CICATRIZATI'ON OF WOUNDS.

V. NEW MATHEMATICAL EXPRESSION OF CICATRIZATION.

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A new formula with two equations gives in mathematical terms Carrel and Hartmann's law:1 "the rate of cicatrization diminishes at the same time as the size but less rapidly."

In the time $dt$, the area cicatrized, $dS$, is proportional to $S$:

$$\frac{dS}{S} = -\lambda \ dt \quad \text{or} \quad \frac{dS}{dt} = -\lambda S$$

$\lambda$ is positive and the formula shows that the rate, $\frac{dS}{dt}$, decreases with $S$.

By integration we get

$$\int \frac{dS}{S} = -\lambda \int dt$$

(1) \hspace{1cm} \log S = -\lambda t + \log S_0 \quad \text{or} \quad S = S_0 e^{-\lambda t}$

where $S_0$ is the initial area.

If the coefficient $\lambda$ is constant, the law of cicatrization can be expressed by simple logarithmic formula.

The rate of cicatrization decreases less rapidly than the size; that is, $\lambda$ is not constant and must increase slightly when the area decreases. In the time $dt$ the variation of $\lambda$, $d\lambda$, is proportional to $\lambda$:

$$d\lambda = \mu \lambda dt \quad \text{or} \quad \frac{d\lambda}{dt} = \mu \lambda$$

If $\mu$ is positive, the equation indicates that $\lambda$ increases because the derivative $\frac{d\lambda}{dt}$ is positive.

By integration we get
\[ \int \frac{d\lambda}{\lambda} = \mu \int dt \]
(2) \[ \log \lambda = \mu t + \log \lambda_0 \quad \text{or} \quad \lambda = \lambda_0 e^{\mu t} \]

where \( \lambda_0 \) is the initial value of \( \lambda \). \( \lambda \) is calculated by equation (2) and with this value of \( \lambda \) we can obtain \( S \) by the equation (1).

The two coefficients \( \lambda_0 \) and \( \mu \) may be determined to make the values calculated and observed correspond.

The area at any time can be obtained immediately without calculating the intermediate areas.