ON THE DISTRIBUTION AND DIRECTION OF MOTION
OF THE INTERFERENCE BANDS OF LIGHT
FORMED BY THIN PLATES AS THE
THICKNESS OF THE PLATE
VARIES.*

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PLATE 33.

When taking records with the micrograph1 it was observed that
in some instruments the direction of motion of the interference
bands of light for an increase of air pressure was opposite to the
direction of motion in other instruments. In some the diameters
of the circular bands contracted as the two interference plates ap-
proached each other, while in others they expanded. Ordinary
spectacle lenses were used, and the facts are explained by supposing
that the surfaces were curved instead of being plane. It will be
shown that there is a critical value of the curvature of the inter-
ference surfaces, and when the curvature is greater than the critical
value, the bands move in one direction, but when less, they move in
the opposite direction.

A photograph of interference bands formed between two pieces
of plate glass (eight by ten inches), the upper resting upon the
lower by its own weight, is reproduced in figure 1, and shows two
characteristic kinds of areas that usually appear when two pieces
of common glass are placed together. The first kind of area, A.
is surrounded by bands in closed curves, which often approximate
an elliptical shape. The second kind of area, B, is characterized

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1 The Micrograph. An Instrument Which Records the Microscopic Move-
ments of a Diaphragm by Means of Light Interference and Some Records of
Physiological Events Showing the Registration of Sound Waves Including the

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by there being no bands surrounding it, each band radiating off indefinitely, and by the system of bands resembling pairs of conjugate hyperbolae.

Each of these two classes of areas has another distinguishing feature which is not manifest when the plates are at rest. As the two plates are pressed more firmly together the whole system of bands moves, and those surrounding the area, A, may contract or they may expand as the plates approach each other. If they contract, we may call the curvature of the plates at this point positive, and if they expand, negative. Assuming the area A to be positive, it is evident from the photograph that the hyperbolic bands on the right and left of the area B recede from each other,
while the pair above and below approach each other. The reverse would be true at B if the area A had a negative curvature.

Consider the case of two plates of glass which are exactly parallel, as in text-figure 1. The plates may be imagined to be infinite in extent, and the source of illumination to be pure monochromatic light uniformly distributed over the whole upper hemisphere. Let the observer’s eye be located at E, a distance, \( p \), above the interference surfaces. The observer looking at any point, \( P \), of these surfaces sees light by reflection from one point of the source only. Whether or not there is destructive interference at this point \( P \) may be determined by the formula:

\[
\cos R = \frac{n\lambda}{2\mu e}. \tag{1}
\]

\( R \) = angle of refraction \( = PP'U = PE'O \).
\( n \) = any integer.
\( \lambda \) = wave-length of light.
\( \mu \) = refractive index of the material of the plate.
\( e \) = thickness of the air between the two glass plates.

If the equation is satisfied at the point \( P \), there will be total interference, and a dark band will appear at this part of the plates. From symmetry the same values obtain at all points on the circumference of a circle with \( O \) as center and \( OP \) as radius.

As the point \( P \) approaches the point \( O \), the angle \( R \) decreases and its cosine increases, and since all the quantities in the right hand member of the equation are constant for a given plate and light source, except \( n \), which is any integer, the equation will not be satisfied again as \( P \) approaches \( O \) until \( \cos R \) has increased sufficiently to admit a value of \( n \) one unit greater than it had at \( P \). The next consecutive band, therefore, corresponding to a value of \( n \) one unit greater, appears as a circle with \( OP \), less than \( OP \), as radius. The system of bands seen by the observer at \( E \), therefore, consists of a set of concentric circles centered at \( O \), which extend from an infinite distance, where \( R = 90^\circ \), \( \cos R = 0 \), and \( n = 0 \), to the center, \( O \), where \( R = 0 \), \( \cos R = 1 \), and therefore,

\[
n_o = \frac{2\mu e}{\lambda}. \tag{2}
\]

Since the thin plate is in this case supposed to be air, \( \mu = 1 \), and

\[
n_o = \frac{2}{\lambda} e. \tag{3}
\]

The total number of bands, \( n_o \), constituting the system is, therefore, proportional to the thickness of the plate. When this thickness, \( e \), is equal to one half wave-length of the light, there is but one band in the whole field, and in general the total number of bands composing the system is equal to the number of half wave-lengths of light in the thickness of the plate.
The set of curves shown in text-figure 2 is obtained from equation (1) by putting $\mu = 1$, and writing,

$$ R = \cos^{-1} \frac{n\lambda}{2e}. $$

The abscissae give the angles, $R$, from $0^\circ$ to $90^\circ$, and the ordinates the thickness of plate, $e$, in half wave-lengths of light for successive values of the integer $n$, from $n=0$, to $n=20$. The set of curves well illustrates the effects produced by separating or approximating the two plates; that is, changing the value of $e$.

**Text-Fig. 2.** A series of curves representing the required curvature of the upper interference surface, the lower being plane, to produce total interference or darkness over its entire area for any given value of the integer $n$, the curves being given for all values of $n$ from 0 to 20. The curves JK indicate the diameters of the circles of the first ring, counting from the center outward just as the central ring is appearing or disappearing.

If a horizontal line is drawn at a fixed height, say where $e = 16$ half wave-lengths, this line is tangent to the sixteenth curve at $O$, and cuts the lower curves successively at the points $P_{16}, P_{14}, P_{13}$, etc., indicating that with a plate of thickness sixteen, the sixteenth band is just appearing at the center, $O$, when the fifteenth, fourteenth, thirteenth, etc. bands have radii equal to $OP_{16}, OP_{14}, OP_{13}$, etc., or about $21^\circ, 28.5^\circ$, etc. If the upper plate is raised and the lower remains stationary, the effect on the bands is seen by raising the horizontal line. It
becomes tangent to and then cuts successively the curves corresponding to increasing values of \( n \), and the two points of intersection of this line with any given curve recede from each other, indicating expanding bands, or, vice versa, contracting bands, for an approximation of the parallel plates. Parallel plates would, therefore, give a positive instead of a negative area at \( A \), figure 1.

Suppose the lower plate remains plane, and the upper becomes curved—convex toward the plane plate—and is raised at the same time and made to coincide in its upward motion successively with the curves in text-figure 2; in such a case the whole surface would become dark each time a curve is passed in its upward course, because every point of this surface satisfies equation (4) for some value of \( n \).

Imagine a plane surface, say at the height \( e = 16 \), to be gradually bent from its plane, as \( OQ \), until it comes into coincidence with the curve \( OS \). Evidently the sixteenth band at the common point of tangency, \( O \), will remain undisturbed, and the fifteenth band will recede or expand as the point \( T \) advances until it passes entirely off the plate. If the bending of the surface be continued beyond the curve \( OS \) to some position as \( OU \), then the seventeenth band appears at an infinite distance and advances to the point \( V \). This curve, \( OU \), shows at a glance that the point \( V \) approaches \( O \) as the plate is separated more and more from the plane plate below; that is, the whole system of circular bands in this case contracts with a separation of, and expands with an approach of the plates. We have called such a system of bands negative when referring to figure 1.

On the other hand, before the plate is bent sufficiently to coincide with the curve \( OS \), as at \( OQ \) for example, it is evident that the point \( T \) recedes as the plate is raised, showing expanding bands for a separation of the plates. This corresponds to a positive curvature, as above defined.

The curve \( OS \) has, therefore, a critical radius of curvature because the direction of motion of the bands for approaching plates changes sign from contraction to expansion. To find the radius of curvature at the center, \( O \), of the upper plate, equation (4) may be expressed in terms of the radii, \( OP = x \), along the plate, and the length of the perpendicular, \( p \), from the eye to the plate, thus,

\[
\frac{4}{n^2\lambda} + \frac{1}{p^2} = \frac{a^2}{b^2} - 1.
\]

In the following, the thickness of the upper glass plate is considered to be small as compared to the distance, \( p \), from the eye to the plate, so that \( p \) is practically constant for all positions of the point \( P \). Considering \( e \) and \( x \) to be the variables and all other quantities constant, equation (5) represents an hyperbola with semi-major axis \( a = n\lambda/2 \), and semi-minor axis \( b = p \). Since the radius of curvature at the apex of an hyperbola, corresponding to the point \( O \), is \( R = b/a \), we have

\[
R = \frac{2p\lambda}{n\lambda}.
\]

The set of curves in text-figure 3 is obtained from (5) by assuming the distance from the eye to the plate, \( p \), as 30 cms. One hyperbola is drawn for each value
of \( n \) from 0 to 20 as in text-figure 2. The curve \( JK \) gives the radius in centimeters of the first band when counting in the reverse order from the center outward, and shows that with parallel plates the diameter of the first band diminishes as the plates separate. Substituting \( p = 30 \text{ cms.} \), the radius of curvature in (6) becomes,

\[
R = \frac{1800}{n\lambda^2}.
\]

Assuming that the wave-length of the particular light used is \( 40.47 \times 10^{-4} \text{ cms.} \),

**Text-Fig. 3.** This set of curves is similar to that in text-figure 2 except that the distance \( p \) from the eye to the plate (see text-figure 1) is assumed to be 30 cms., and the abscissae are given in centimeters instead of in terms of the angle \( R \) in degrees.

we have the following critical radii of curvature for the corresponding values of \( n \):

<table>
<thead>
<tr>
<th>( n )</th>
<th>( R )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>445. kilometers</td>
</tr>
<tr>
<td>10</td>
<td>44.5 kilometers</td>
</tr>
<tr>
<td>100</td>
<td>4.45 kilometers</td>
</tr>
<tr>
<td>1000</td>
<td>0.445 kilometers</td>
</tr>
</tbody>
</table>
Since by (4), at the origin of the curves where \( x = 0 \), we have \( e_o = \frac{\lambda}{2} \), the values of \( n \) above signify the number of half wave-lengths' thickness of the plate at the center, \( O \). In the last case where \( n = 1000 \), the separation of the plates is about four tenths of a millimeter when the critical radius of curvature is nearly half a kilometer.

The same equation (5) may be expressed in a different way, giving the set of curves in text-figure 4 by considering the thickness of the plate, \( e \), to remain constant while the variables are \( x \) and \( n \). The transformed equation may be written,

\[
x = \frac{1}{\frac{1}{\sqrt{\frac{4e^2}{\lambda^2} - 1}}}
\]

and represents an equation of the fourth degree for these variables.

In general, when two plates of glass are superposed to produce bands as in figure 1, there appear two kinds of critical points or areas, the elliptical and the hyperbolic, as at \( A \) and \( B \) respectively. The elliptical kind may be either positive or negative, and the surface for either positive or negative has a
definite radius of curvature. The case when the two surfaces are concave to
each other and the center is a point of maximum separation would be a case
of positive area; when the surfaces are more convex to each other than the
critical value, the center is a point of minimum separation and the area is
negative.

The second class of critical area, $B$, differs from $A$ in there being neither a
maximum nor a minimum point, but a critical point of flexure. The two lines
taking the direction of the asymptotes to the hyperbolae are approximately
parallel with the lower plate, and the curvature of the surfaces between the two
asymptotes on opposite sides of the center is of the same kind, both being
either positive or negative. The curvature on adjacent sides of one asymptote
is opposite, being positive on one side and negative on the other. The asympto-
totes are themselves the loci of points having a maximum gradient, the surface
angle with the parallel plate being here a maximum.

TWO SOURCES OF LIGHT.

There is another interesting effect seen in the photograph, figure 1. The
bands $P_1, P_2, P_3$ appear not to be focused as sharply as the other bands. These
blurred bands occur at regular intervals of thirteen bands apart. The explana-
tion is that the picture was not taken with pure monochromatic light, but with
a mercury vapor lamp without the use of screens. The spectrum of this light
has four prominent bright lines, one in yellow, green, blue, and violet, respectively.

The wave-lengths of the two latter colors are: blue, $43.59 \times 10^{-6}$ cms., and
violet, $40.47 \times 10^{-6}$ cms. Commercial Seed's plates were used, and they were
not sensitized for the yellow and green lines of the spectrum. The picture
shows, however, that both of the lines of shorter wave-length affected the plate,
one more than the other.

If two wave-lengths, $\lambda_1$ and $\lambda_2$, are employed, and the corresponding numbers
of the bands counting from an infinite distance to a point, $R$, of the plate be
denoted by $n_1$ and $n_2$ respectively, we have from (1),

$$
\cos R_1 = \frac{n_1 \lambda_1}{2\pi c} \quad \text{and} \quad \cos R_2 = \frac{n_2 \lambda_2}{2\pi c},
$$

from which we obtain the relation, when $R_1 = R_2$,

$$
\frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1}.
$$

In the case of the mercury vapor lamp, using wave-lengths $43.59$ and $40.47$ cms.,
we have,

$$
\frac{\lambda_2}{\lambda_1} = 1.077 = \frac{14}{13} \quad \text{approximately.}
$$

Hence,

$$
\frac{n_1}{n_2} = \frac{14}{13} = \frac{28}{26} = \frac{42}{39} = \text{etc.}
$$

The first dark band corresponding to $n = 0$ occurs at an infinite distance for
all wave-lengths, but the next point where the dark bands coincide for these
particular wave-lengths is thirteen bands for the one and fourteen bands for the other, counting from an infinite distance toward the center. Succeeding coincidences will occur at all multiples of 13 and 14 until the center, $O$, is reached.

At these points of coincidence of the dark bands, the picture should be sharp with strong contrast, but half way between these positions the blurred effect should be most pronounced. If the actinic effect of the two sources upon the plate is very unequal, the blurred region should be confined to a smaller area than it would be were the two sources equally intense. The fact that this effect is confined to one or two bands shows that the two sources in the mercury vapor lamp have an unequal effect upon the plate.

A complete explanation of all the points considered arising from the interference of light between two glass plates, as seen in figure 1, has thus been obtained from an application of the very simple formula (1).

EXPLANATION OF PLATE 33.

Fig. 1. A photograph of light interference bands obtained with two pieces of plate glass, eight by ten inches, and a mercury vapor light, the plates being observed by reflected light. The areas A and B are each typical of those observed when two common pieces of glass are used. Note that the bands appear indistinct at regular intervals of thirteen bands $P_1, P_2, P_3$. This is explained by the character of the light and the kind of photographic plate used.